Calculus 2E Assignment - Semester 2, 2000 CE

Joseph Curtis;

Question 1

$$\frac{d^2y}{dx^2} = x^2(x-1)^3(x-3)$$

For inflexion points; $\frac{d^2y}{dx^2} = 0$

$$x^2(x-1)^3(x-3) = 0$$

$$x^2 = 0$$
$$x = \sqrt{0}$$
$$x = 0$$

$$(x-1)^3 = 0$$
$$x - 1 = \sqrt[3]{0}$$
$$x - 1 = 0$$
$$x = 1$$

$$x - 3 = 0$$
$$x = 3$$

Points of inflexion at x = 3, x = 1, x = 0

When $x = \frac{1}{2}, y'' = \frac{5}{64}$

$x = 2, y'' = -4$

$x = 4, y'' = 432$

$x = -1, y'' = -32$

Range of concave up: 0 < x < 1, x > 3

Range of concave down: 1 < x < 3, x < 0

Question 2

$$y = \frac{3x}{x^2 + 1}$$

For y intercept, set x to zero

$$y = \frac{0}{1} = 0$$

y intercept = 0

For x intercept, set y to zero

$$0 = \frac{3x}{x^2 + 1}$$

$$0 = 3x$$

$$x = 0$$

$$x^2 + 1 \neq 0$$

x intercept = 0

$$y' = \frac{vu' - uv'}{v^2}$$

$$u = 3x$$

$$v = x^2 + 1$$

$$u' = 3$$

$$v' = 2x$$

$$y' = \frac{3(x^2 + 1) - 3x(2x)}{(x^2 + 1)^2}$$

$$y' = \frac{3x^2 + 3 - 6x^2}{(x^2 + 1)(x^2 + 1)}$$

$$y' = \frac{3(x^2+1) - 6x^2}{(x^2+1)(x^2+1)}$$

$$y' = \frac{3-6x^2}{x^2+1}$$

For stationary points y’ = 0

$$\frac{3-6x^2}{x^2+1} = 0$$
$$3-6x^2 = 0$$
$$3 = 6x^2$$
$$x^2 = \frac{1}{2}$$
$$x = \pm\sqrt{\frac{1}{2}}$$
x = ± 0.707

Stationary points at x = 0.707 and x = -0.707, and y = 5.24 and –5.24 respectively

$$y'' = \frac{vu' - uv'}{v^2}$$

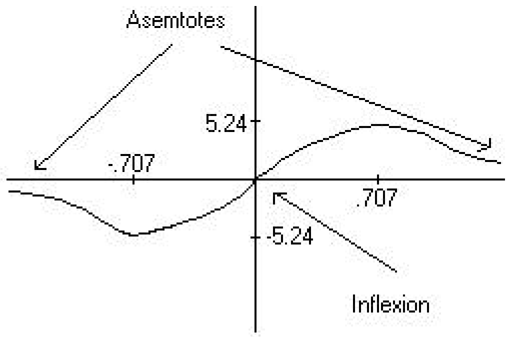
$$u = 3-6x^2$$
$$v = x^2+1$$
$$u' = -12x$$
$$v' = 2x$$
$$y'' = \frac{-12x(x^2+1) - 2x(3-6x^2)}{(x^2+1)(x^2+1)}$$
$$y'' = \frac{-12x^3 - 12x - 6x + 12x^3}{(x^2+1)(x^2+1)}$$

$$y'' = \frac{-18x}{(x^2+1)^2}$$

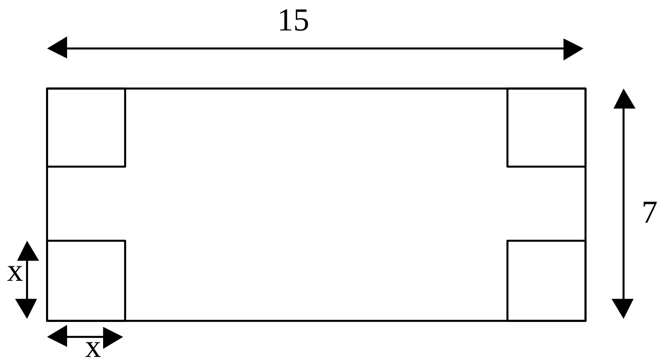
For inflexion points y’’ = 0
$$\frac{-18x}{(x^2+1)^2} = 0$$

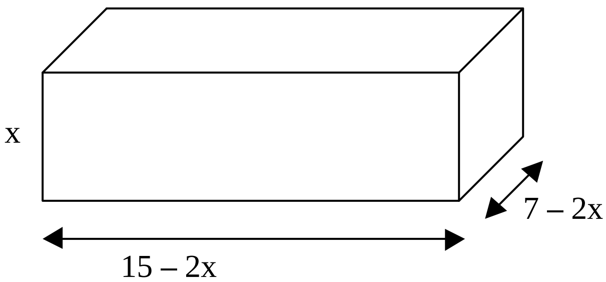
$-18x = 0$

$x = 0$



Question 3





$v = x(15-2x)(7-2x)$

For maximum: v' = 0

v'' = negative

$v = 15x - 2x^2(7-2x)$

$v = 105x - 30x^2 - 14x^2 + 4x^3$

$v = 4x^3 - 44x^2 + 105x$

$v' = 12x^2 - 88x + 105$

To find x for minimum

$12x^2 - 88x + 105 = 0$

$6x \quad -3,-35$

$2x \quad -35,-3$

$0 = (6x - 35)(2x - 3)$

$6x - 35 = 0$

$6x = 35$

$x = 5\frac{5}{6}$

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

To see if max or min
For max v’’ = negative

For $5\frac{5}{6}$

$v'' = 24x - 88$

$v'' = 52$

For $\frac{3}{2}$

$v'' = -52$

The box has a maximum volume of $72u^3$ for the value of x = $\frac{3}{2}$

Question 4

For minimum surface area (s) s’ = 0, s’’ = positive.
v = 63π cm^3

$v = \pi r^2 h$

$63\pi = \pi r^2 h$

$h = \frac{63\pi}{\pi r^2}$

$h = \frac{63}{r^2}$

$$s = 2pr^2 + 2pr(h+1)$$
$$s = 2pr^2 + 2pr(63r^{-2} + 1)$$
$$s = 2pr^2 + 126pr^{-1} + 2pr$$
$$s = 2pr^2 + 2pr + 126pr^{-1}$$

$$s' = 4pr + 2p - 126pr^{-2}$$

For max or min, s' = 0

$$4pr + 2p - 126pr^{-2} = 0$$

Cannot factorize, using factor theorem to check for factors for f(r)

f(1) = -25.13
f(2) = -67.54
f(3) = 0 – factor
f(4) = 31.80
f(5) = 53.28

r – 3 = 0

Find other factors by dividing by (r – 3)

$$\begin{array}{r|l} & 4p + 14pr^{-1} - 42pr^{-2} \\ \hline r-3 & 4pr + 2p - 126pr^{-2} \end{array}$$
$$4px - 12p$$
$$14p$$
$$14p - 42pr^{-1}$$
$$-42pr^{-1} - 126pr^{-2}$$
$$-42pr^{-1} - 126pr^{-2}$$

$$(r-3)(4p + 14pr^{-1} - 42pr^{-2})$$

Cannot factorize $(4p + 14pr^{-1} - 42pr^{-2})$

$\therefore r - 3$ is the only factor
$\therefore$ surface area is min when $r = 3$

$$h = \frac{63}{r^2}$$

$$h = \frac{63}{9}$$
$$h = 7$$

$$s'' = 4\pi + 252\pi r^{-3}$$
$$s'' = 41.88$$
s’’ is positive $\therefore$ surface area is minimum.

Radius = 3, height = 7

$$s = 2\pi r^2 + 2\pi r(h+1)$$
$$s = 18\pi + 6\pi \times 8$$
$$s = 18\pi + 48\pi$$
$$s = 66\pi$$

A maximum surface area of 66π cm^2 or 207.345 cm^2 is achieved with a radius of 3cm and a height of 7cm.